Mathematical Predictions of Gas Permeability for Automotive Air-Bags

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ABSTRACT: The mathematical relationship combining an applied pressure drop and the resultant gas velocity through a woven fabric is important to the air-bag industry to predict the performance of new materials before they are woven. The main difficulty in formulating a mathematical solution is the complexity of the woven fabric structure. In available publications, fabric pores had normally been represented as a series of cylindrical pipes. This article considers the same approach and analyzes some of the equations to review their industrial applicability. Because none of the equations have been found adaptable in predicting the permeability behavior of air-bag fabrics, experimental data have been used to generate empirical equations. The data were generated using a dynamic air permeability tester that was used to project air at high pressure through a variety of air-bag fabrics. A static permeability tester was also used to generate results through the same fabrics at lower pressures. The final equations combine the fabric cover factor and the pressure differential to give the resultant gas velocity. © 2000 John Wiley & Sons, Inc. J Appl Polym Sci 77: 2104–2112, 2000

Key words: air-bag; nylon 6.6; woven; cover factor; fabric pores; pressure differential; gas velocity; empirical; predictive; power law

INTRODUCTION AND BACKGROUND

The original automobile air-bag was developed in the early 1970s by General Motors in the United States. The company fitted about 11,000 vehicles with driver and passenger air bags from 1973 to 1976.[1] This short life was due to a lack of public interest, and at this time, customers were unwilling to pay the extra price for the devices to be fitted. Since then, further research was conducted on air-bag performance and a much improved system was introduced to the market in the late 1980s. Now, the global air-bag industry is worth hundreds of millions of dollars and most people consider an air-bag to be a standard safety item in a new car.

Largely, automobile air-bags are manufactured from nylon 6.6 filament yarns but nylon 6, nylon 4.6, and polyester yarns are also used. The yarns are used to make plain-weave fabrics, which are particularly dense and hard wearing. Until recently, most fabrics were coated with a thin layer of neoprene or silicone rubber because these fabrics were extremely good at containing the hot gas that was produced when the air-bag was deployed. Moreover, after a crash, the hot gas was expelled through vents at the rear of the bag that channeled it away from the occupant, thereby minimizing the risk of burns. However, the shortcomings of coated air-bags are their excessive thickness, inability to be folded into small spaces, and the tendency to degrade over time. These characteristics preclude the bags from being folded into highly styled vehicle interiors and the degradation can lead to catastrophic

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failure when the bag is deployed. The coated air-bag is also considered to be environmentally unfriendly.

The answer has been to develop the uncoated air-bag, which, until recently, had only been found in the passenger side of the car. The uncoated air-bag releases the gas through the fabric pores, and for this reason, it must be deployed at a relatively lower pressure with a cooler gas. The fabrics are designed to release the gas in a controlled manner to cushion the occupant appropriately and to prevent burns when the gas permeates through the fabric. At first, these bags were placed in the passenger side of the car where the operating conditions were less demanding. Now, however, they are being fitted in the driver compartment where the operating conditions are more severe. These conditions are the result of a shorter inflation time, a higher pressure inside the bag, and greater forces that are generated when the bag bursts from its housing. In air-bag development, therefore, the ability to predict air-bag fabric permeability, particularly at high pressure, is highly advantageous and this is the subject of the article.

THEORY OF AIR PERMEABILITY THROUGH WOVEN STRUCTURES

The spaces open to airflow through a woven fabric are notoriously difficult to define and, historically, this has made the task of modeling woven fabric permeability, mathematically particularly difficult. The difficulties arise from the complexities that are introduced by fiber and yarn variables and fabric geometry parameters. Air that is projected at a woven fabric has three possible paths: The first is through the fabric pores, the second is between the filaments that make the yarns, and the third is through the microscopic voids within the filaments themselves.

The only measurement that is relatively easy to quantify is the dimension of the fabric pores, and to overcome their complex dimensions, they have previously been represented as a series of straight cylindrical pipes. Their diameters have been estimated with a calculation or a suitable experimental technique.[2] The length of these pipes has normally been represented by the fabric thickness.

Importantly, this simplification has allowed authors to model woven fabric permeability with equations from fluid dynamics. An equation that is commonly referred to in this context is the Hagen–Poiseuille equation which is given as eq. (1). It is seen that it combines the radius, R, and length, l, of a pipe, the fluid viscosity, μ, and the differential pressure, Δp. It yields the resultant fluid velocity, V.[3] This is a linear equation and, consequently, describes flow where the air velocity is proportional to the pressure drop. This type of flow is uniform and steady and commonly referred to as viscous flow. It excludes any disturbances that may reduce or increase its speed, by a nonproportional amount:

$$V = \frac{R^2 \Delta p}{8\ \mu l} \tag{1}$$

In practical applications, however, this pattern of airflow is difficult to achieve. The air must be projected at exceptionally low pressures through comparatively thick materials with relatively straight pores.[4,5] This combination of factors is rare and clearly does not apply to the high-pressure projection of air through thin, densely woven air-bag fabrics.

The other theoretical predictions in the literature rely upon Bernoulli's law. This equation differs from the Hagen–Poiseuille equation because it describes turbulent flow which is characteristically random and unsteady. Equation (2) is the version of the Bernoulli law used in an investigation by Robertson.[6,7] Terms in this equation include the gas velocity, V, and the gas pressure, h. The other terms are A_0, which is the cross section of the pipe, and g, which is the acceleration due to gravity:

$$V^2 = \frac{2gh}{1 - A_0^2} \tag{2}$$

The problem with using the Bernoulli Law in this application, however, is that it is derived according to a number of basic assumptions, which the irregular pattern of flow through a woven textile structure is unlikely to adhere to. For reference, they stated that (i) viscous effects are negligible, (ii) the flow is steady, and (iii) the flow is incompressible.[3] Indeed, it has been found that since these assumptions are not very well adhered to the equation tends to overestimate the speed of the airflow through a woven material.

As a result of the shortfalls of the two previous approaches, others have begun their studies to model airflow through woven fabrics by quantifying the pattern of flow more precisely. Using a well-established method, Kulichenko and Van-Langenhove compared Reynolds numbers and friction factors derived using permeability data gained in tests on a variety of textile materials.[8] They showed that airflow through textiles was more likely to be transitional. This type of flow is not completely turbulent or completely viscous and is best described as a mixture of the two conditions. Investigators agree that this is probably the most common pattern of flow through woven textile structures.

Thus, to model transitional airflow, investigators have commonly used equations that have a linear term for the viscous element of the flow and a squared term to account for the turbulent part. An equation that epitomizes this approach is the Hagen–Poiseuille equation with the Hagenbach modification. A version of this expression was used by Rainard[9] and is given as eq. (3). The terms in this equation are F_a, the air velocity; R, the radius of the fabric pores; L, the length of the pores; and n, the number of pores across a specified area. The term ρ is the fluid density, η is the fluid viscosity, and m is a constant:

$$\frac{m\rho F_a^2}{\pi^2 R^4 n^2} + \frac{8L\eta}{\pi R^4 n} F_a = \Delta p \qquad (3)$$

The problem that is incurred when applying eq. (3) is the choice of the term m whose value is determined by the pattern of the flow carried along a pipe after the application of the pressure differential. It is based on the shape that streamlines assume during the flow. Initially, the flow lines assume a flat cross section, but after the fluid has traveled a distance denoted the entrance length, the overall effect is a flow profile shaped like a parabola. If this condition is adhered to, m can vary between 2.16 and 2.41.[10]

The difficulty in modeling the pattern of flow through fabric pores, however, is that, due to their exceptionally short length, the flow is unable to develop the parabolic profile. For this reason, the values for the constant m are not suitable and calculated results are considerably higher than those given experimentally.

The overall finding from this investigation, therefore, was that none of the equations from the literature were suitable to predict the speed of air through the air-bag fabrics. The existing equations were unable to predict the air speeds generated through relatively short pipes at the high-pressure drops that are applied in air-bag deployment. The other difficulty in applying the models in industrial applications would be quantifying the dimensions and number of pores across a fabric sample. Accordingly, this study has modeled experimental air permeability results empirically.

EXPERIMENTAL

The experimental work was carried out using static and dynamic permeability testers. This was considered to be appropriate because, together, the two approaches can describe the performance of the fabrics at high and low pressure. A brief description of the two instruments is given below:

Dynamic Testing

The dynamic air permeability tester was built by Textest AG and introduced to the market in 1995.[11] The tester is named the "Airbag Tester" because it is capable of projecting air at fabrics at differential pressures up to 200 kPa, which are analogous to the conditions experienced in air-bag deployment.

The instrument is table-mounted and can be fitted with a range of interchangeable test heads that hold 100, 200, 400, 800, or 1600 cm^3 of gas. Inside each test head, there are two chambers that are connected by a high-speed valve. The upper chamber is a storage area that is filled with gas until it reaches the machine-selected start pressure. The gas is held here until it has reached ambient temperature, which is 20°C. At this point, the high-speed valve is removed and air or another gas is projected at the fabric, which is firmly clamped underneath. In an earlier article,[12] it was reported that the pressure differential attained when the air penetrated the fabric was significantly lower than the start pressure selected on the machine. The magnitude of this pressure drop depended upon the start pressure, the volume of the test head fitted to the machine, and the structure of the fabric, which was tested. The article suggested that to maintain continuity during testing it was essential to choose the start pressure carefully to generate the same penetration pressure in every test.

Table I Specifications of Experimental Fabrics

Yarn Dtex and Type	Filaments per Yarn	End Count (/10 cm)	Pick Count (/10 cm)	Cover Factor Range
700, DuPont T728	105	160	95–160	0.670–0.794
470, DuPont T725	68	160	100–180	0.588–0.715
312, DuPont T[a]	Not available	220	160–220	0.715–0.769
235, DuPont T725	34	260	180–260	0.682–0.764

[a] Experimental yarn.

In this study, the dynamic tester was used to develop fabric-penetrating pressures from 80 to 200 kPa that increased in increments of 20 kPa. These pressure differentials were developed across a range of air-bag fabrics by choosing the correct test heads and machine-selectable start pressures. The largest test heads were needed to generate the lowest pressure drops across the most open fabrics. Altogether, each fabric was tested five times at every penetration pressure. To ensure the continuity of the results, the penetration pressures were never allowed to deviate more than 5 kPa either side of the value that had been selected for a particular set of results.

Following a test, the important measurement from this apparatus was the air velocity that was achieved at the fabric-penetration pressure. This measurement was extrapolated from the tester once the data had been transferred to a computer. In this format, the data were inspected graphically and showed the relationship between the differential pressure and the air velocity over the entire test cycle.

This graphical curve was also used to quantify the relationship between the pressure difference and the gas velocity in the testing. The relationship was described by a parameter named the exponent whose numerical value can vary from about 0.7 to 1.5.[11] These values correlate with the shape of the graphical curve, and if it is less than 1, this curve will be digressive. Alternatively, the testing can generate an exponent that is greater than 1 and the graphical curve corresponding to this performance is progressive. The final scenario is an exponent that is equal to 1 and then the pressure and air speed is proportional and can be described with a straight line.

Static Testing

In addition to the dynamic tests at high-pressure differentials, the air-bag fabrics were also subjected to testing with a static instrument that generated pressures in the order of 500 Pa. This style of testing exemplifies the test standard DIN 53887,[13] which is currently used to quantify the permeability of many commercial air-bag fabrics.

The static testing in this study was performed on an instrument that was also built by Textest AG and named the FX 3300 Air Permeability Tester.[11] The instrument comprised a suction fan and a means of measuring the gas speed generated through the fabric. During the testing, a fabric was laid on a flat test bed, which was furnished with a circular opening. The test head was mounted on an arm directly above the circular opening, and to perform a test, it was depressed onto the specimen. At this point, the instrument adjusted the pressure automatically to derive a steady test pressure across the fabric. For the purposes of this work, the instrument was used to generate fabric-penetrating pressures ranging from 100 to 2999 Pa, which represented its full working range.

Materials

The experimental work was conducted on a range of commercial air-bag fabrics, which were supplied by DuPont Nylon UK. The fabrics were constructed from nylon 6.6 filament yarns with a range of yarn linear densities and a variable number of picks. The yarns were woven according to a plain-weave design. A number of the fabric specifications are given in Table I.

RESULTS

In the early stages of data generation from the dynamic apparatus, it became apparent that the relationship between test pressure and air velocity was rarely linear. This was exemplified by the fact that in the majority of individual tests the value of the exponent was less than 1.

A closer inspection of the graphical relationship between the pressure drop and the resultant air velocities showed that the best fit to the experimental data was by a power-law equation. This is a trend that has been described elsewhere in the literature.[11,14]

If the relationship between air pressure and velocity can be defined by a power-law equation, it follows that if the results are converted to log values on a linear scale they will lie on or near to a straight line. To test the hypothesis, the pressure differentials and air velocities from this work were converted to log values, then plotted graphically. Since the preliminary study indicated that the data did lie on straight lines, the original hypothesis was true. Thereafter, the data were analyzed using a linear regression program to define these straight lines mathematically. The LINEST regression program in Microsoft Excel was used for the analysis.

The important parameters given by the program were the values of E_{exp}, which were the gradients of the straight lines, and the values of k, which were the intercepts on the y axis. These two values were used as components of the basic power law given as eq. (4) and in combination quantified the permeability performance of each air-bag fabric. The other terms in the equation were the air velocity, AP, and the pressure differential, p:

$$AP = kp^{E_{\mathrm{exp}}} \tag{4}$$

Further values that were collected during the regression analysis were the R^2 values associated with each of the straight lines. These results were valuable because they indicated how well the straight lines fitted the data. R^2 values are calculated by comparing the experimental data to the equivalent readings given by the straight-line equation. A perfect fit to the data is given by an R^2 value equal to 1.

The R^2 values that described the straight lines in this analysis were all in the region of 0.99. They confirmed that the permeability results could be described accurately by the straight lines that had been superimposed over the original data.

The values of E_{exp} were normally less than 1 and confirmed that the relationship between the pressure and permeability through these particular air-bag fabrics was nearly always digressive. This feature was most prominent through the open-weave fabrics. The larger E_{exp} values tended to be associated with the materials with higher cover factors. This trend was described in an earlier article by the same authors.[15] The article suggested that the exponent values were strongly influenced by the extension of the fabric and open area available to the air at the point of gas penetration.

Inspection of the k values that were derived using the regression program showed that as the fabric cover factor rose the k values tended to fall. The trend can be explained if this parameter is interpreted as the permeability of the fabric at a very low pressure differential. As the fabric cover rises, the airflow is stifled and the values get progressively lower.

DEVELOPMENT OF A THEORETICAL EQUATION TO PREDICT FABRIC PERMEABILITY

In the preliminary work, the values of E_{exp} and k were correlated with the significant fabric structural characteristics that are listed below from (i) to (iv). The work indicated that the clearest correlations were achieved with the parameters that were relatively easy to derive and the best parameter was the fabric cover factor. The benefits of using the fabric cover factor to describe air-bag fabrics in an empirical equation were twofold: First, it did not require extensive experimental work to gain its values and, second, it accounted for other fabric characteristics such as the yarn linear density and the end-and-pick count:

(i) fabric cover factor;
(ii) fabric pick density (yarns per 10 cm);
(iii) fabric pore size (μm);
(iv) fabric thickness (mm).

Predicting the Value of E_{exp}

The correlation between the fabric cover factor and all the E_{exp} values derived with the permeability data led to the graph depicted in Figure 1. The graph illustrates that all the values lie on and around a straight line. To account for the variation on either side of the straight line, the

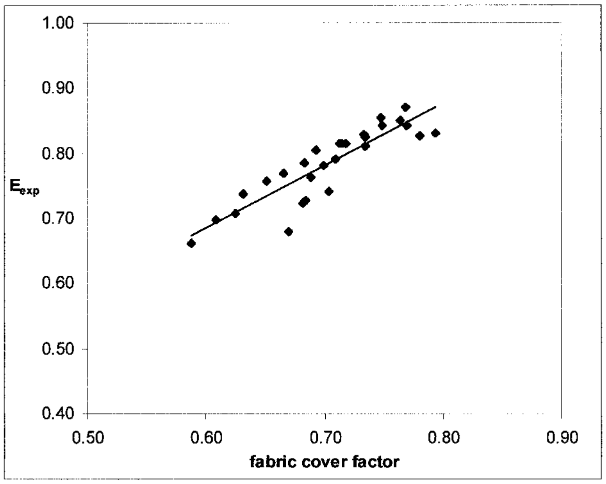

Figure 1 Relationship between fabric cover factor and values of E_{exp}.

same data were input into the regression program to give the numerical degree of variation denoted by s_{ey}. Incorporating this value meant that the straight-line equation was based on the framework of eq. (5). The numerical interpretation of the straight line in Figure 1 is given by eq. (6), where the term k_f is the fabric cover factor. The final version allows upper and lower values of E_{exp} to be generated from the same equation:

$$E_{\text{exp}} = (mk_f + c) \pm (2 \times s_{\text{ey}}) \tag{5}$$

$$E_{\text{exp}} = 0.954k_f + 0.113 \pm (2 \times 0.0271) \tag{6}$$

Predicting the Value of *k*

To derive an equation to predict the values of k, these values were also plotted against the fabric cover factors, k_f. This time, the graphical relationship between the two different parameters was more difficult to identify, and in the preliminary work, the data were superimposed with trend lines formed with eqs. (i)–(iii):

(i) $1/x$;
(ii) power;
(iii) exponential.

The work showed that the power and exponential trend lines both fitted very accurately. The superimposed curves for the 700-, 470-, 312-, and 235-dtex fabrics could all be described by R^2 values in the order of 0.9. However, the power trend was selected for this analysis to maintain continuity with the earlier observations between pressure and air velocity.

Figure 2 shows the relationship between the values of k and the fabric cover factors. The graph illustrates that the values of k related to all the air-bag fabrics could not be fitted with a single trend line. The trend line on the left covers the points derived from the testing on the fabrics made from the 700-, 470-, and 312-dtex yarns. The trend line over the 235-dtex results lies some distance to the right. The reason is that this second trend line is believed to be linked to the unknown finishing routines that the materials were subjected to when they were manufactured.

The equation for the curve encompassing the 700-, 470-, and 312-dtex points is given as eq. (7). The R^2 value for this trend line was 0.945, which indicated that the trend line provided a good fit to the experimental data. The second curve was described by eq. (8) and encompasses the points related to the materials made from the 235-dtex yarns. This curve had an R^2 value equal to 0.940, which also indicated a accurate fit to the original data:

$$k_{(700,470,312)} = 0.00001344k_f^{-10.65} \tag{7}$$

$$k_{(235)} = 0.00000314k_f^{-17.32} \tag{8}$$

Theoretical Equations

The final empirical relationships to predict the permeability performance of the woven air-bag

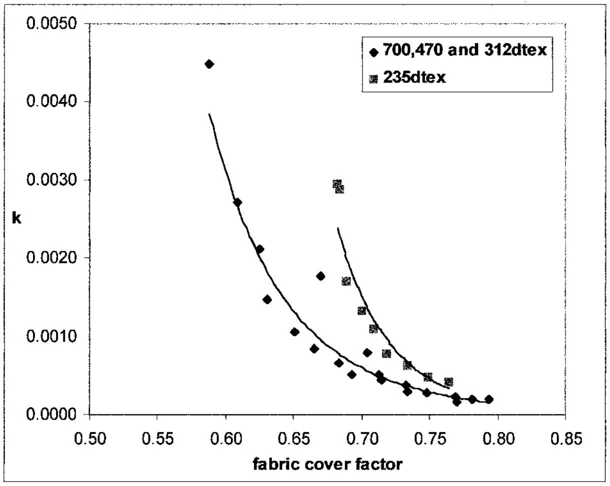

Figure 2 Relationship between fabric cover factor and values of *K*.

fabrics were eqs. (9) and (10). Equation (9) was derived to predict the permeability of air-bag fabrics made from 700-, 470-, and 312-dtex yarns. The expression uses eq. (6) to give $E_{\exp}$ and eq. (7) to derive the value of k for a fabric with a particular fabric cover factor, k_f:

$$AP_{(700,470,312)} = 0.00001344 k_f^{-10.65} p^{(0.954k_f+0.113)\pm(2\times 0.0271)} \quad (9)$$

Equation (10) was formulated specifically to calculate the permeability of air-bag materials made from 235-dtex yarns. This equation acknowledges the difference in the permeability data that was exhibited in Figure 2. The components of this expression are eqs. (6) and (8) that give the $E_{\exp}$ and k, values, respectively:

$$AP_{(235)} = 0.00000314 k_f^{-17.32} p^{(0.954k_f+0.113)\pm(2\times 0.0271)} \quad (10)$$

Accuracy of the Empirical Equations

To determine the accuracy of the empirical equations, they were used to predict the permeability of some of the experimental fabrics. This involved comparing the predicted and experimental air speeds that were generated through each of the air-bag fabrics specified in Table I at a pressure difference of 50 kPa.

The first results that were considered were associated with the air-bag fabrics made from the relatively coarse 700-dtex nylon 6.6 yarns. The graph in Figure 3 correlates the fabric cover factors with the experimental results and the upper and lower predicted results calculated with eq. (9). It is seen from the graph that in this instance, for all the fabrics, the predicted results were an overestimation of the experimental air speeds. Despite this trend, however, the lower predictions showed a high level of accuracy and did not exceed the experimental results by any more than 0.5 m/s.

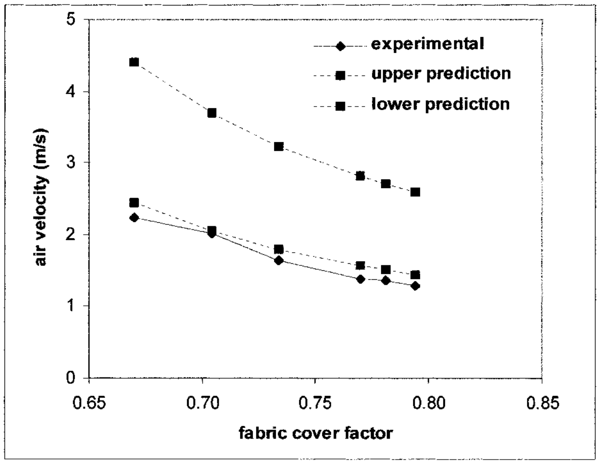

Figure 3 Experimental and predicted air velocities generated at 50 kPa on the 700-dtex air-bag fabrics.

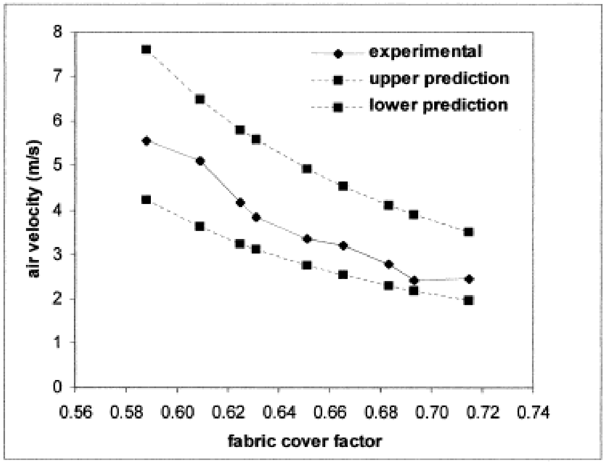

Figure 4 Experimental and predicted air velocities generated at 50 kPa on the 470-dtex air-bag fabrics.

The same comparison was made between the experimental and predicted air speeds generated through the air-bag fabrics made from 470-dtex yarns. The outcome of this analysis is seen in Figure 4, which shows that, in this instance, the experimental results were encapsulated by the predicted readings. The graph indicates that the accuracy of the two sets of predicted results was clearly affected by the fabric cover factor, but for the majority of fabrics, the most accurate prediction was the lower result. The encouraging aspect of the graph was that the contours of the lines joining the experimental and predicted points were clearly similar.

In the same analysis, eq. (9) was employed to predict the permeability of the air-bag materials made from 312-dtex yarns. The results that relate to the permeability performance of these fabrics at the 50-kPa pressure difference are given in Figure 5. The most marked difference between this correlation and those in Figures 3 and 4 was that, in this instance, the upper predictions from the equation were nearest to the experimental results. The other significant trend from the graph was that the accuracy of the upper results from the equation improved slightly as the fabric cover increased.

In the last part of the comparative assessment, eq. (10) was used to predict the permeabilities of the fabrics whose yarn specification was 235 dtex. The experimental and predicted air speeds that were generated at a pressure difference of 50 kPa are given in Figure 6. Again, the graph shows that the accuracy of the predicted results depended on the cover factor of the air-bag fabric whose performance was being considered. For these particular air-bag materials at the lower cover factors, the lower predicted results were the most accurate. In comparison, for the materials with the higher cover factors, the upper results from the equation were closer to the original experimental data.

CONCLUSIONS

The study has revealed that air-bag fabric permeability cannot easily be described by equations that define fabric pores as straight cylindrical pipes. The equations in the literature do not apply to high-pressure projections of air through very short pipes and they often need measurements to describe the fabric structure that must be found by additional experimental work.

The relationship between pressure and air velocity through an air-bag fabric is normally nonlinear. The relationship is described more accurately by a curve based on the power law. This is true, because when permeability results for individual fabrics are converted to log values and then plotted graphically, they lie on and around straight lines. Thus, in this format, permeability data can be entered into a linear regression program to obtain the values of E_{exp} and k that describe the gradient and intercept of the same straight lines.

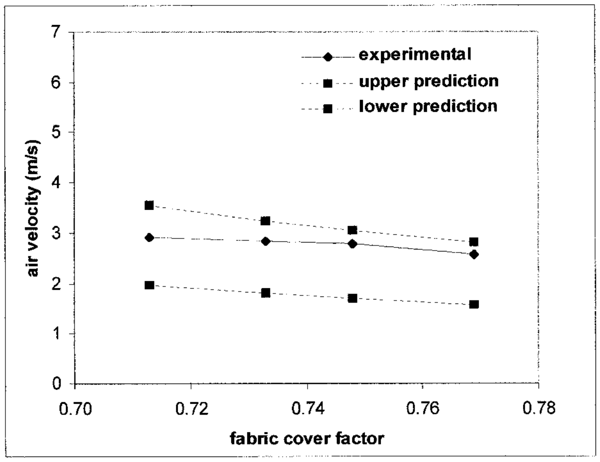

Figure 5 Experimental and predicted air velocities generated at 50 kPa on the 312-dtex air-bag fabrics.

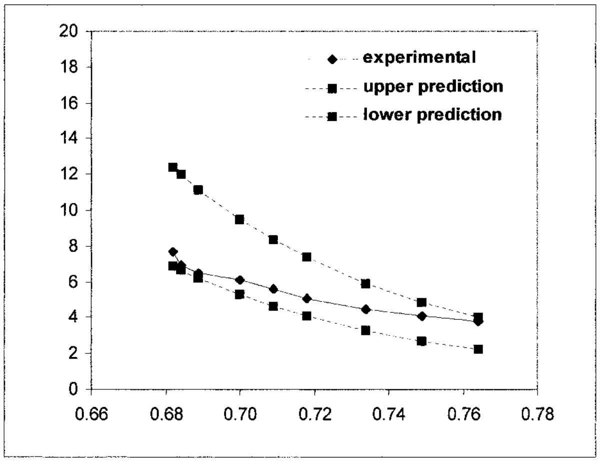

Figure 6 Experimental and predicted air velocities generated at 50 kPa on the 235-dtex air-bag fabrics.

The empirical equations to describe the permeability of the air-bag fabrics were generated by plotting the values of E_{exp} and k against the corresponding fabric cover factors. The correlation between the values of E_{exp} and the fabric cover factor yielded a single straight line. The values of k were superimposed with two trend lines based on power equations. The final empirical equations had broad upper and lower bands that enveloped most of the experimental data. The accuracy of the results was seen to depend on the specification of the yarns and the cover factor of the fabric whose permeability was being predicted.

Finally, it is envisaged that in the future these equations will be improved by including more experimental permeability data from a wider range of fabrics. This should enable the development of a more generalized equation to predict the air permeability of air-bag fabrics.

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